paper, and called a *focometer*. It has been constructed to the author's designs by Messrs. Nalder Brothers, to whom sundry of the mechanical details are due.

The paper also describes the results obtained with the focometer upon various lenses, some of them being microscope objectives, others camera lenses. The author finds in several of these lenses that the principal planes are crossed: the distance between the symmetric points being less than four times the focal length. In some other lenses which are achromatic in respect of bringing all rays to a common principal focus, the positions of the principal planes are different for rays of different colours. In one lens, a microscope objective by Reichert, the principal planes are not only crossed but are actually at a greater distance apart than the two principal foci. The paper is accompanied by a sheet of full-size drawings showing the construction of the instrument and its details.

IV. "The Numerical Registration of Colour. Preliminary Note." By Captain W. de W. Abney, C.B., R.E., D.C.L., F.R.S.

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The Committee of the Royal Society on Colour Vision having put into my hands the determination of the colour of certain signal glasses, a memorandum was drawn up on the method of the numerical registration of colours and submitted to them. They considered that it should be submitted to the Royal Society, and having slightly modified it, it is presented as a preliminary note of a part of a paper which will be subsequently submitted by General Festing and myself as Part III of "Colour Photometry."

It must be premised that a colour is determined when its hue, its purity, and its luminosity are known, the last constant being its comparison with the white light before its passage through a transparent coloured body, or with white light reflected from a white surface if it be an opaque coloured body such as a pigment.

There has hitherto been a certain amount of difficulty on the part of normal-eyed persons in stating the exact hue of compound colours in terms of any standard; in fact, I believe, except by the method given in the Second Part of "Colour Photometry" ('Phil. Trans.,' A, 1888), there has been no exact means indicated of reproducing a colour from measurements made. The method which will be described can take the place of the previous plan for certain purposes, more particularly when it is the impression on the eye which has to be considered. Any colour can be reproduced from the registration numbers with the greatest exactness.

To persons who are totally colour-blind to one sensation, viz., the

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green or the red, the matching of a compound colour with a simple one in the spectrum should possess no difficulties. Taking the trichromatic theory of three sensations for the normal-eyed person, it is evident that only the following classes of sensations are possible in the normal-eyed, the green colour-blind, and the red colour-blind :—

Normal eye.	Green colour-blind.	Red colour-blind.
Red	Red	—
Green	—	Green.
Violet	Violet..........	Violet.
Mixtures of red and green	—	—
Mixtures of red and violet......	Mixtures of red and violet.	—
Mixtures of green and violet	—	Mixtures of green and violet.
Mixtures of red, green, and violet	—	—

If we take as a type of colour-blindness the green colour-blind person, we see that every colour in the spectrum must be either red, violet, or these colours mixed with more or less white light, since these two sensations when excited in certain proportions give the sensation of white. At one place, which is commonly called the neutral point, these proportions are such that there is the impression of white light ; it follows that, between this neutral point and each end of the spectrum, the rays are mixtures of violet and white or red and white, the dilution of the colours varying from no white to all white. As every compound colour must be a mixture of the same two colours in certain proportions, it follows that the green colour-blind person can match every compound colour with some one ray of the spectrum, and that every colour must to him be either red or violet, diluted with different proportions of white light.

In the same way, a person who is colour-blind to the red can match any colour with a single spectrum colour, and he will see it as green or violet diluted with more or less white light. This can be readily understood, but it is not quite so plain how any colour sensation felt by the normal eye can be referred to the spectrum.

The following is an outline of the reasoning which leads up to the method of registration employed :—

If we take three rays in the spectrum—one in the red between C and the red lithium line, which we will call *R*, another in the green between F and *b*, which we will call *G*, and a third in the violet near G, but on the H side of it, and which we may call *V*—then, by varying their intensities (which is equivalent to varying the luminosities) and mixing them, we can give the same impression to the eye that any compound colour gives, and that of any intermediate simple spectrum colour but very slightly diluted with white light. With these same three colours, but in different proportions, we can also give the impression of white light to the eye. The intermediate spectrum colours between the green and the violet rays selected, when slightly diluted, are imitated by mixing these rays together in different proportions, and similarly those lying between the red and the green by mixing together these rays in different proportions—and there is some ray present in the spectrum which, when very slightly diluted with white light, has the same colorific effect on the eye as the mixtures of the pairs V and G and G and R in any proportions whatever.

Let the luminosities of the rays R, G, and V, which give the impression of white light, be a, b, and c units respectively, and p, q, and r those which give that of the colour which has to be registered and reproduced. We then get the following equations—where W is white, w its luminosity, Z the colour, and z its luminosity—

$$aR+bG+cV=wW \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(i)};$$

$$pR+qG+rV=zZ \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(ii)}.$$

Then evidently—

$$(a+b+c)=w, \quad \text{and} \quad (p+q+r)=z.$$

Let $p=\alpha a$, $q=\beta b$, $r=\gamma c$.
Then we may write (ii) as—

$$\alpha aR+\beta bG+\gamma cV=zZ \quad \ldots\ldots\ldots\ldots \quad \text{(iii)}.$$

Now, either α, β, or γ must be smaller than the other two. As an example, if α be the smallest, we multiply (i) by α, when we get—

$$\alpha aR+\alpha bG+\alpha cV=\alpha wW \quad \ldots\ldots\ldots\ldots \quad \text{(iv)}.$$

Subtracting (iv) from (iii), we get—

$$(\beta-\alpha)\,bG+(\gamma-\alpha)\,cV=zZ-\alpha wW.$$

Now, it has already been stated that between V and G there is some ray which gives the same sensation of colour, mixed with a very small quantity of white light, as the above mixture of V and G—let us call it X and its luminosity x [x being evidently equal to $(\beta-\alpha)b+(\gamma-\alpha)c$], and μ the luminosity of the small quantity of white added.

We then get $zZ=xX+(\mu+\alpha)\,W$.

Here we have the colour Z in terms of a single ray, and of white light.

This same holds good when in (ii) γ is smaller than α and β; but it does not do so should it happen that β is the smallest, for there is no part of the spectrum which contains simple colours giving the same sensation to the eye as mixtures of red and blue. There is, however, a very simple way in which the registration of such a colour (which it must be remarked must be of a purple tone) can be effected. It can be fixed by its complementary. To do this we must add to (ii) a certain amount of R and V, which will make the whole white. Thus, suppose in (iii) α to be larger than γ and γ than β, then we must add $\phi b\mathrm{G}+\theta c\mathrm{V}$, and we have—

$$\alpha aR+(\beta+\phi)\,bG+(\gamma+\theta)\,cV=nW=Z+\phi bG+\theta cV;$$

but $(\beta+\phi)$ and $(\gamma+\theta)$ each equal α; $\therefore n=\alpha w$;

$$\therefore Z+\phi bG+\theta cV=\alpha wW.$$

Now, between V and G in the spectrum there is some single colour which gives the sensation of the mixture of G and V. Let it be X' with luminosity x', together with white, whose luminosity is μ', which equals $(\phi b+\theta c)$.

$$\therefore Z+x_1X'+\mu'W=\alpha wW;$$

$$Z=(\alpha w-\mu')\,W-x'X';$$

which again is the colour expressed in terms of white light less the complementary colour. We have thus arrived at the very simple deduction that the hue and luminosity of any colour, however compounded, may be registered by a reference to white light and a single ray of the spectrum.

In practice this dominant ray is very easy to find. Suppose we wish to determine numerically the colour of a signal-green glass in the electric light; we should proceed as follows:—

The colour-patch apparatus described in the Appendix to the Bakerian Lecture "On Colour Photometry" ('Phil. Trans.,' 1886, Abney and Festing) is employed, and the coloured glass is placed between the silvered mirror, which reflects the beam already reflected from the first surface of the first prism of the spectrum apparatus, and the screen, and a square image of that surface of the prism, showing the tint of the glass, is formed on the screen by means of the lens. Touching this image is a square patch of white light, formed by the re-combination of the spectrum by means of another lens. An opaque slide, containing an adjustable slit, is moved across the spectrum in the manner described in the paper referred to, until the colour of this last patch is approximately the same hue as that of the glass.

In the path of the reflected beam, but between the prism and the silvered mirror, is inserted a piece of plain glass, which can be made to reflect part of the white beam into the monochromatic patch of light, a square patch of this white light being formed by means of a third lens. We thus have monochromatic light mixed with white light. The requisite intensity of the added white light can be adjusted by means of rotating sectors which open and close at will during rotation, and the integrated luminosity of the mixed beams can be altered by this, together with the adjustable slit in the slide. The slit may probably have to be moved in the spectrum to make the hue of these mixed lights the same as that of the glass, but by trial the position of the ray, whose colour, when diluted with white light, makes the match, is readily found. The position of the slit in the spectrum is noted, as also the aperture of the sectors. The relative luminosities of the beam reflected from the plain glass mirror and of the coloured ray are next measured by placing a rod in the path of the two beams, and equalising by the sectors the luminosity of the shadows which are illuminated, the one by the spectral ray, and the other by the white light. When the sector aperture is noted the registration is complete, as far as hue is concerned, but the luminosity of the ray transmitted through the glass should be compared with that of the unabsorbed reflected beam, and then the total luminosity is doubly recorded.

Should the colour of a pigment be in question, the ray reflected from the silvered mirror is made to fall on the pigmented surface, and the same procedure adopted.

Should a purple glass (say) have to be registered, we proceed in a slightly different manner, the patch of colour light passing through the purple glass is superposed over the spectrum patch, and the slit in the slide is moved till a ray is found which will make white light when superposed on the colour of the glass. The luminosities of this white light, of the reflected beam, and of the spectral colour, are compared *inter se*, and there are then sufficient data on which to make numerical registration. In the paper which will be submitted to the Royal Society, and of part of which this is a preliminary note, the details of registration will be entered into more fully.

The signal glasses having to be used at night with oil or gas, their hue must be registered in these lights. As the spectrum colours are always the same, it is convenient to use the electric light spectrum, and the only alteration in the apparatus is to use two gas lights to illuminate two square apertures, in front of one of which the glass, whose colour has to be measured, is placed. The images of these apertures are thrown on the screen, the coloured image touching the square image of the spectral colour patch, and the second image over the latter. The same determinations are gone through as those just determined.

The following are the determinations of some of the coloured glasses submitted to the Committee, recorded in this manner:—

Glass.	Electric light.			Gas light.		
	Dominant wave-length.	Percentage of white light in colour.	Luminosity, naked light = 100.	Dominant wave-length.	Percentage of white light in colour.	Luminosity, naked light = 100.
Great Western ruby glass	6250	7	10·4	6275	0	13·1
L.B.S.C.	6200	0	10·4	6200	12	13·0
Great Northern	6250	0	9·0	6275	0	10·0
Great Western signal green............	4925	46	21·8	5070	50	18·1
L.B.S.C.	4925	38	16·2	5050	34	12·5
Great Northern	5100	61	19·2	5170	62	19·4
Great Eastern	5000	54	15·0	5120	40	15·0
Saxby and Farmer's, as ordinarily supplied where no special glass ordered	4925	24	7·6	5050	22	6·9
Bottle green glass (District Railway)	5500	32	9·1	5320	50	10·6
Cobalt blue	4675	38	4·4	4650	59	3·3

The following are determinations of some coloured pigments in the electric light:—

Colours.	Dominant wave-length.	Percentage of white light.	Percentage of luminosity (white paper = 100).
Vermilion	6100	2·5	14·8
Emerald green	5220	59·0	22·7
F ultramarine	4720	61·0	4·4
Brown paper	5940	50·0	25·0
,, (greyer)	5670	67·0	19·5
Orange	5915	4·0	62·5
Chrome yellow	5835	26·0	77·7
Blue-green	5005	42·5	14·8
Eosin dye ('Sporting Times')	6400	72·0	44·7
Cobalt	4820	55·5	14·5

The determination of the colours in Maxwell's disks by this plan enables them to be used much more effectively than if they are simply indeterminate colours. For since the sum of the luminosities of any colours is equal to the luminosity of the same colours integrated, it follows that when using the disks the colours of the dominant wave-lengths are really mixed and the white light inherent in each case can be deducted.

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